

Topic 3

Be only

1998 Calculus BC Free-Response Questions

6. A particle moves along the curve defined by the equation $y = x^3 - 3x$. The x -coordinate of the particle, $x(t)$, satisfies the equation $\frac{dx}{dt} = \frac{1}{\sqrt{2t+1}}$, for $t \geq 0$ with initial condition $x(0) = -4$.
- (a) Find $x(t)$ in terms of t .
 - (b) Find $\frac{dy}{dt}$ in terms of t .
 - (c) Find the location and speed of the particle at time $t = 4$.
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END OF EXAMINATION

1999

The College Board
Advanced Placement Examination

CALCULUS BC

SECTION II

Time—1 hour and 30 minutes

Number of problems—6

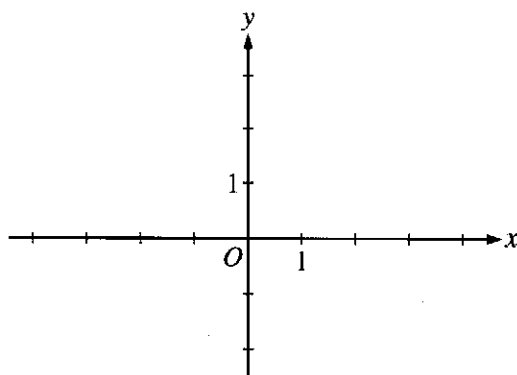
Percent of total grade—50

REMEMBER TO SHOW YOUR SETUPS AS DESCRIBED IN THE GENERAL INSTRUCTIONS.

1. A particle moves in the xy -plane so that its position at any time t , $0 \leq t \leq \pi$, is given by

$$x(t) = \frac{t^2}{2} - \ln(1 + t) \text{ and } y(t) = 3 \sin t.$$

- (a) Sketch the path of the particle in the xy -plane below. Indicate the direction of motion along the path.



- (b) At what time t , $0 \leq t \leq \pi$, does $x(t)$ attain its minimum value? What is the position $(x(t), y(t))$ of the particle at this time?
- (c) At what time t , $0 < t < \pi$, is the particle on the y -axis? Find the speed and the acceleration vector of the particle at this time.
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GO ON TO THE NEXT PAGE 

2000 AP® CALCULUS BC FREE-RESPONSE QUESTIONS

**CALCULUS BC
SECTION II, Part B**

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.

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4. A moving particle has position $(x(t), y(t))$ at time t . The position of the particle at time $t = 1$ is $(2, 6)$, and the velocity vector at any time $t > 0$ is given by $\left(1 - \frac{1}{t^2}, 2 + \frac{1}{t^2}\right)$.
- (a) Find the acceleration vector at time $t = 3$.
 - (b) Find the position of the particle at time $t = 3$.
 - (c) For what time $t > 0$ does the line tangent to the path of the particle at $(x(t), y(t))$ have a slope of 8?
 - (d) The particle approaches a line as $t \rightarrow \infty$. Find the slope of this line. Show the work that leads to your conclusion.
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5. Consider the curve given by $xy^2 - x^3y = 6$.

(a) Show that $\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}$.

- (b) Find all points on the curve whose x -coordinate is 1, and write an equation for the tangent line at each of these points.
- (c) Find the x -coordinate of each point on the curve where the tangent line is vertical.
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2001 AP® CALCULUS BC FREE-RESPONSE QUESTIONS

CALCULUS BC
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.

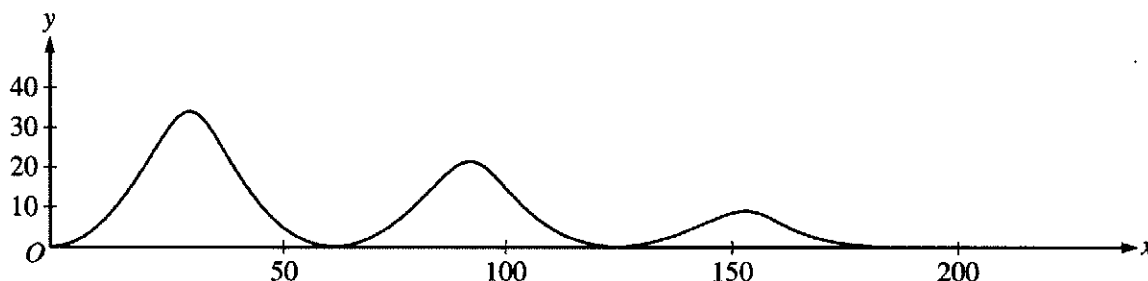
1. An object moving along a curve in the xy -plane has position $(x(t), y(t))$ at time t with

$$\frac{dx}{dt} = \cos(t^3) \text{ and } \frac{dy}{dt} = 3 \sin(t^2)$$

for $0 \leq t \leq 3$. At time $t = 2$, the object is at position $(4, 5)$.

- (a) Write an equation for the line tangent to the curve at $(4, 5)$.
 - (b) Find the speed of the object at time $t = 2$.
 - (c) Find the total distance traveled by the object over the time interval $0 \leq t \leq 1$.
 - (d) Find the position of the object at time $t = 3$.
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2002 AP[®] CALCULUS BC FREE-RESPONSE QUESTIONS



3. The figure above shows the path traveled by a roller coaster car over the time interval $0 \leq t \leq 18$ seconds. The position of the car at time t seconds can be modeled parametrically by

$$x(t) = 10t + 4 \sin t$$

$$y(t) = (20 - t)(1 - \cos t),$$

where x and y are measured in meters. The derivatives of these functions are given by

$$x'(t) = 10 + 4 \cos t$$

$$y'(t) = (20 - t) \sin t + \cos t - 1.$$

- (a) Find the slope of the path at time $t = 2$. Show the computations that lead to your answer.
- (b) Find the acceleration vector of the car at the time when the car's horizontal position is $x = 140$.
- (c) Find the time t at which the car is at its maximum height, and find the speed, in m/sec, of the car at this time.
- (d) For $0 < t < 18$, there are two times at which the car is at ground level ($y = 0$). Find these two times and write an expression that gives the average speed, in m/sec, of the car between these two times. Do not evaluate the expression.

END OF PART A OF SECTION II

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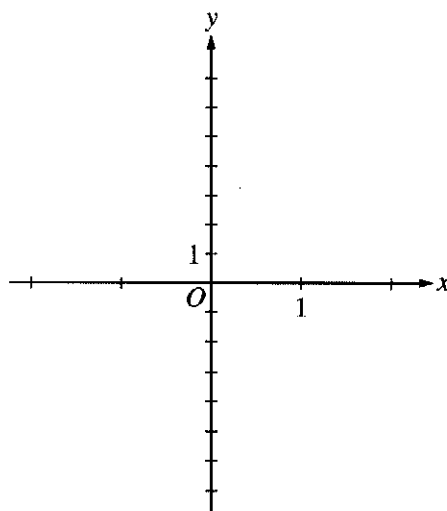
CALCULUS BC
SECTION II, Part A

Time—45 minutes

Number of problems—3

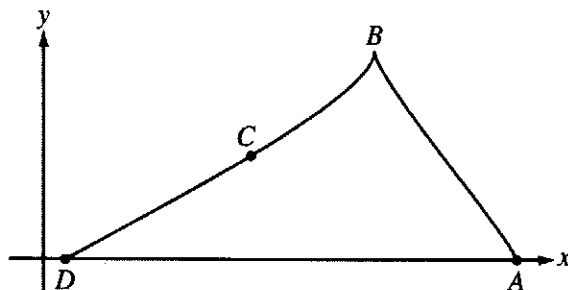
A graphing calculator is required for some problems or parts of problems.

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1. A particle moves in the xy -plane so that its position at any time t , for $-\pi \leq t \leq \pi$, is given by $x(t) = \sin(3t)$ and $y(t) = 2t$.
- (a) Sketch the path of the particle in the xy -plane provided. Indicate the direction of motion along the path.
(Note: Use the axes provided in the test booklet.)



- (b) Find the range of $x(t)$ and the range of $y(t)$.
- (c) Find the smallest positive value of t for which the x -coordinate of the particle is a local maximum. What is the speed of the particle at this time?
- (d) Is the distance traveled by the particle from $t = -\pi$ to $t = \pi$ greater than 5π ? Justify your answer.
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2. A particle starts at point A on the positive x -axis at time $t = 0$ and travels along the curve from A to B to C to D , as shown above. The coordinates of the particle's position $(x(t), y(t))$ are differentiable functions of t , where $x'(t) = \frac{dx}{dt} = -9\cos\left(\frac{\pi t}{6}\right)\sin\left(\frac{\pi\sqrt{t+1}}{2}\right)$ and $y'(t) = \frac{dy}{dt}$ is not explicitly given. At time $t = 9$, the particle reaches its final position at point D on the positive x -axis.
- At point C , is $\frac{dy}{dt}$ positive? At point C , is $\frac{dx}{dt}$ positive? Give a reason for each answer.
 - The slope of the curve is undefined at point B . At what time t is the particle at point B ?
 - The line tangent to the curve at the point $(x(8), y(8))$ has equation $y = \frac{5}{9}x - 2$. Find the velocity vector and the speed of the particle at this point.
 - How far apart are points A and D , the initial and final positions, respectively, of the particle?
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2003 AP[®] CALCULUS BC FREE-RESPONSE QUESTIONS (Form B)

CALCULUS BC
SECTION II, Part B

Time—45 minutes

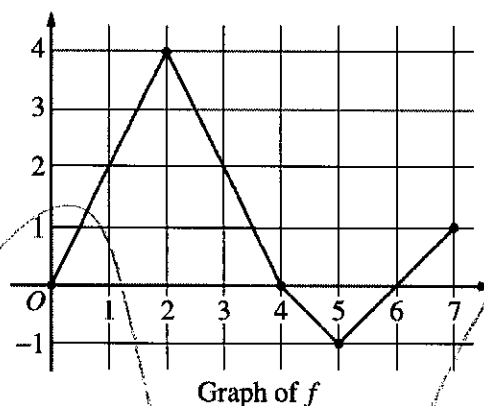
Number of problems—3

No calculator is allowed for these problems.

4. A particle moves in the xy -plane so that the position of the particle at any time t is given by

$$x(t) = 2e^{3t} + e^{-7t} \text{ and } y(t) = 3e^{3t} - e^{-2t}.$$

- Find the velocity vector for the particle in terms of t , and find the speed of the particle at time $t = 0$.
- Find $\frac{dy}{dx}$ in terms of t , and find $\lim_{t \rightarrow \infty} \frac{dy}{dx}$.
- Find each value t at which the line tangent to the path of the particle is horizontal, or explain why none exists.
- Find each value t at which the line tangent to the path of the particle is vertical, or explain why none exists.



5. Let f be a function defined on the closed interval $[0, 7]$. The graph of f , consisting of four line segments, is shown above. Let g be the function given by $g(x) = \int_2^x f(t) dt$.

- Find $g(3)$, $g'(3)$, and $g''(3)$.
- Find the average rate of change of g on the interval $0 \leq x \leq 3$.
- For how many values c , where $0 < c < 3$, is $g'(c)$ equal to the average rate found in part (b)? Explain your reasoning.
- Find the x -coordinate of each point of inflection of the graph of g on the interval $0 < x < 7$. Justify your answer.

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3. An object moving along a curve in the xy -plane has position $(x(t), y(t))$ at time $t \geq 0$ with $\frac{dx}{dt} = 3 + \cos(t^2)$.

The derivative $\frac{dy}{dt}$ is not explicitly given. At time $t = 2$, the object is at position $(1, 8)$.

- (a) Find the x -coordinate of the position of the object at time $t = 4$.
 - (b) At time $t = 2$, the value of $\frac{dy}{dt}$ is -7 . Write an equation for the line tangent to the curve at the point $(x(2), y(2))$.
 - (c) Find the speed of the object at time $t = 2$.
 - (d) For $t \geq 3$, the line tangent to the curve at $(x(t), y(t))$ has a slope of $2t + 1$. Find the acceleration vector of the object at time $t = 4$.
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END OF PART A OF SECTION II

2004 AP[®] CALCULUS BC FREE-RESPONSE QUESTIONS (Form B)

CALCULUS BC
SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.

1. A particle moving along a curve in the plane has position $(x(t), y(t))$ at time t , where

$$\frac{dx}{dt} = \sqrt{t^4 + 9} \text{ and } \frac{dy}{dt} = 2e^t + 5e^{-t}$$

for all real values of t . At time $t = 0$, the particle is at the point $(4, 1)$.

- (a) Find the speed of the particle and its acceleration vector at time $t = 0$.
- (b) Find an equation of the line tangent to the path of the particle at time $t = 0$.
- (c) Find the total distance traveled by the particle over the time interval $0 \leq t \leq 3$.
- (d) Find the x -coordinate of the position of the particle at time $t = 3$.

2. Let f be a function having derivatives of all orders for all real numbers. The third-degree Taylor polynomial for f about $x = 2$ is given by

$$T(x) = 7 - 9(x - 2)^2 - 3(x - 2)^3.$$

- (a) Find $f(2)$ and $f''(2)$.
- (b) Is there enough information given to determine whether f has a critical point at $x = 2$?
If not, explain why not.
If so, determine whether $f(2)$ is a relative maximum, a relative minimum, or neither, and justify your answer.
- (c) Use $T(x)$ to find an approximation for $f(0)$. Is there enough information given to determine whether f has a critical point at $x = 0$?
If not, explain why not.
If so, determine whether $f(0)$ is a relative maximum, a relative minimum, or neither, and justify your answer.
- (d) The fourth derivative of f satisfies the inequality $|f^{(4)}(x)| \leq 6$ for all x in the closed interval $[0, 2]$. Use the Lagrange error bound on the approximation to $f(0)$ found in part (c) to explain why $f(0)$ is negative.

2005 AP[®] CALCULUS BC FREE-RESPONSE QUESTIONS (Form B)

**CALCULUS BC
SECTION II, Part A**

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.

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1. An object moving along a curve in the xy -plane has position $(x(t), y(t))$ at time $t \geq 0$ with

$$\frac{dx}{dt} = 12t - 3t^2 \text{ and } \frac{dy}{dt} = \ln(1 + (t - 4)^4).$$

At time $t = 0$, the object is at position $(-13, 5)$. At time $t = 2$, the object is at point P with x -coordinate 3.

- (a) Find the acceleration vector at time $t = 2$ and the speed at time $t = 2$.
 - (b) Find the y -coordinate of P .
 - (c) Write an equation for the line tangent to the curve at P .
 - (d) For what value of t , if any, is the object at rest? Explain your reasoning.
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WRITE ALL WORK IN THE TEST BOOKLET.

2006 AP[®] CALCULUS BC FREE-RESPONSE QUESTIONS**CALCULUS BC
SECTION II, Part B****Time—45 minutes****Number of problems—3****No calculator is allowed for these problems.**

t (seconds)	0	10	20	30	40	50	60	70	80
$v(t)$ (feet per second)	5	14	22	29	35	40	44	47	49

4. Rocket A has positive velocity $v(t)$ after being launched upward from an initial height of 0 feet at time $t = 0$ seconds. The velocity of the rocket is recorded for selected values of t over the interval $0 \leq t \leq 80$ seconds, as shown in the table above.
- (a) Find the average acceleration of rocket A over the time interval $0 \leq t \leq 80$ seconds. Indicate units of measure.
- (b) Using correct units, explain the meaning of $\int_{10}^{70} v(t) \, dt$ in terms of the rocket's flight. Use a midpoint Riemann sum with 3 subintervals of equal length to approximate $\int_{10}^{70} v(t) \, dt$.
- (c) Rocket B is launched upward with an acceleration of $a(t) = \frac{3}{\sqrt{t+1}}$ feet per second per second. At time $t = 0$ seconds, the initial height of the rocket is 0 feet, and the initial velocity is 2 feet per second. Which of the two rockets is traveling faster at time $t = 80$ seconds? Explain your answer.
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WRITE ALL WORK IN THE PINK EXAM BOOKLET.

2006 AP[®] CALCULUS BC FREE-RESPONSE QUESTIONS

3. An object moving along a curve in the xy -plane is at position $(x(t), y(t))$ at time t , where

$$\frac{dx}{dt} = \sin^{-1}(1 - 2e^{-t}) \text{ and } \frac{dy}{dt} = \frac{4t}{1 + t^3}$$

for $t \geq 0$. At time $t = 2$, the object is at the point $(6, -3)$. (Note: $\sin^{-1} x = \arcsin x$)

- (a) Find the acceleration vector and the speed of the object at time $t = 2$.
 - (b) The curve has a vertical tangent line at one point. At what time t is the object at this point?
 - (c) Let $m(t)$ denote the slope of the line tangent to the curve at the point $(x(t), y(t))$. Write an expression for $m(t)$ in terms of t and use it to evaluate $\lim_{t \rightarrow \infty} m(t)$.
 - (d) The graph of the curve has a horizontal asymptote $y = c$. Write, but do not evaluate, an expression involving an improper integral that represents this value c .
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END OF PART A OF SECTION II

2006 AP[®] CALCULUS BC FREE-RESPONSE QUESTIONS (Form B)

2. An object moving along a curve in the xy -plane is at position $(x(t), y(t))$ at time t , where

$$\frac{dx}{dt} = \tan(e^{-t}) \text{ and } \frac{dy}{dt} = \sec(e^{-t})$$

for $t \geq 0$. At time $t = 1$, the object is at position $(2, -3)$.

- (a) Write an equation for the line tangent to the curve at position $(2, -3)$.
 - (b) Find the acceleration vector and the speed of the object at time $t = 1$.
 - (c) Find the total distance traveled by the object over the time interval $1 \leq t \leq 2$.
 - (d) Is there a time $t \geq 0$ at which the object is on the y -axis? Explain why or why not.
-

WRITE ALL WORK IN THE EXAM BOOKLET.

2007 AP[®] CALCULUS BC FREE-RESPONSE QUESTIONS (Form B)

2. An object moving along a curve in the xy -plane is at position $(x(t), y(t))$ at time t with

$$\frac{dx}{dt} = \arctan\left(\frac{t}{1+t}\right) \text{ and } \frac{dy}{dt} = \ln(t^2 + 1)$$

for $t \geq 0$. At time $t = 0$, the object is at position $(-3, -4)$. (Note: $\tan^{-1}x = \arctan x$)

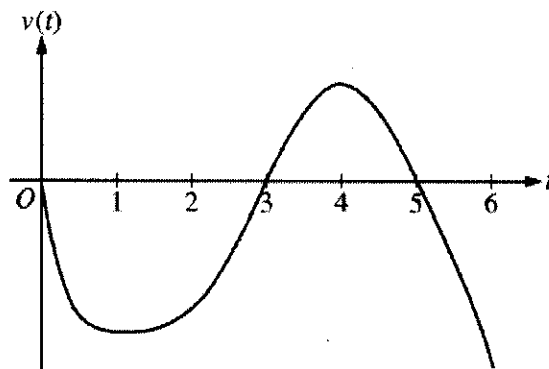
- (a) Find the speed of the object at time $t = 4$.
 - (b) Find the total distance traveled by the object over the time interval $0 \leq t \leq 4$.
 - (c) Find $x(4)$.
 - (d) For $t > 0$, there is a point on the curve where the line tangent to the curve has slope 2. At what time t is the object at this point? Find the acceleration vector at this point.
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WRITE ALL WORK IN THE EXAM BOOKLET.

2008 AP[®] CALCULUS BC FREE-RESPONSE QUESTIONS

CALCULUS BC
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.



Graph of v

4. A particle moves along the x -axis so that its velocity at time t , for $0 \leq t \leq 6$, is given by a differentiable function v whose graph is shown above. The velocity is 0 at $t = 0$, $t = 3$, and $t = 5$, and the graph has horizontal tangents at $t = 1$ and $t = 4$. The areas of the regions bounded by the t -axis and the graph of v on the intervals $[0, 3]$, $[3, 5]$, and $[5, 6]$ are 8, 3, and 2, respectively. At time $t = 0$, the particle is at $x = -2$.
- (a) For $0 \leq t \leq 6$, find both the time and the position of the particle when the particle is farthest to the left. Justify your answer.
- (b) For how many values of t , where $0 \leq t \leq 6$, is the particle at $x = -8$? Explain your reasoning.
- (c) On the interval $2 < t < 3$, is the speed of the particle increasing or decreasing? Give a reason for your answer.
- (d) During what time intervals, if any, is the acceleration of the particle negative? Justify your answer.
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WRITE ALL WORK IN THE PINK EXAM BOOKLET.

2008 AP[®] CALCULUS BC FREE-RESPONSE QUESTIONS (Form B)

**CALCULUS BC
SECTION II, Part A**

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.

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1. A particle moving along a curve in the xy -plane has position $(x(t), y(t))$ at time $t \geq 0$ with

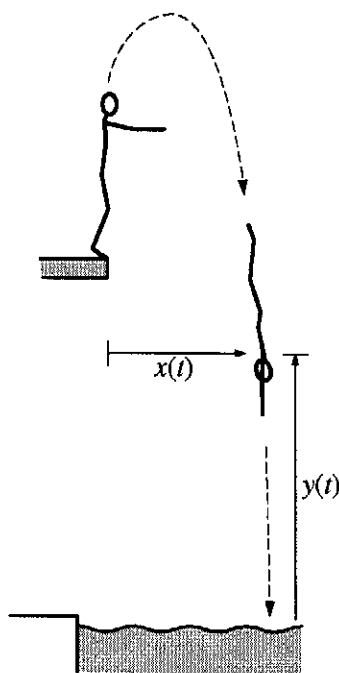
$$\frac{dx}{dt} = \sqrt{3t} \text{ and } \frac{dy}{dt} = 3 \cos\left(\frac{t^2}{2}\right).$$

The particle is at position $(1, 5)$ at time $t = 4$.

- (a) Find the acceleration vector at time $t = 4$.
 - (b) Find the y -coordinate of the position of the particle at time $t = 0$.
 - (c) On the interval $0 \leq t \leq 4$, at what time does the speed of the particle first reach 3.5 ?
 - (d) Find the total distance traveled by the particle over the time interval $0 \leq t \leq 4$.
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WRITE ALL WORK IN THE EXAM BOOKLET.

2009 AP[®] CALCULUS BC FREE-RESPONSE QUESTIONS



Note: Figure not drawn to scale.

3. A diver leaps from the edge of a diving platform into a pool below. The figure above shows the initial position of the diver and her position at a later time. At time t seconds after she leaps, the horizontal distance from the front edge of the platform to the diver's shoulders is given by $x(t)$, and the vertical distance from the water surface to her shoulders is given by $y(t)$, where $x(t)$ and $y(t)$ are measured in meters. Suppose that the diver's shoulders are 11.4 meters above the water when she makes her leap and that

$$\frac{dx}{dt} = 0.8 \quad \text{and} \quad \frac{dy}{dt} = 3.6 - 9.8t,$$

for $0 \leq t \leq A$, where A is the time that the diver's shoulders enter the water.

- Find the maximum vertical distance from the water surface to the diver's shoulders.
- Find A , the time that the diver's shoulders enter the water.
- Find the total distance traveled by the diver's shoulders from the time she leaps from the platform until the time her shoulders enter the water.
- Find the angle θ , $0 < \theta < \frac{\pi}{2}$, between the path of the diver and the water at the instant the diver's shoulders enter the water.

WRITE ALL WORK IN THE PINK EXAM BOOKLET.

END OF PART A OF SECTION II

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